Quantum Mechanical stability of fermion-soliton systems

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Topological objects resulting from symmetry breakdown may be either stable or metastable depending on the pattern of symmetry breaking. However, if they acquire zero-energy modes of fermions, and in the process acquire non-integer fermionic charge, the metastable configurations also get stabilized. In the case of Dirac fermions the spectrum of the number operator shifts by $1/2$. In the case of majorana fermions it becomes useful to assign negative values of fermion number to a finite number of states occupying the zero-energy level, constituting a *majorana pond*. We determine the parities of these states and prove a superselection rule. Thus decay of objects with half-integer fermion number is not possible in isolation or by scattering with ordinary particles. The result has important bearing on cosmology as well as condensed matter physics.

PACS numbers: 11.27.+d, 11.30.Er, 11.30.Fs, 98.80.Cq

INTRODUCTION

Solitons present the possibility of extended objects as stable states within Quantum Field Theory. Although these solutions are obtained from semi-classical arguments in weak coupling limit, their validity as quantal states is justified based on the associated topological conservation laws. A more curious occurrence is that of fermionic zero-energy modes trapped on such solutions. Their presence requires, according to well known arguments[1][2], an assignment of half-integer fermion number to the solitonic states. In the usual treatment, the back reaction of the fermion zero-modes on the soliton itself is ignored. However, the fractional values of the fermionic charge have interesting consequence for the fate of the soliton if the latter is not strictly stable. The reason for this is that if the configuration were to relax to trivial vacuum in isolation, there is no particle-like state available for carrying the fractional value of the fermionic charge. Dynamical stability of such objects was pointed out in [3], in cosmological context in [4], [5] and more recently in [6][7][8]. Fractional fermion number phenomenon also occurs in condensed matter systems and its wide ranging implications call for a systematic understanding of the phenomenon.

The impossibility of connecting half-integer valued states to integer valued states suggests that a superselection rule[9] [10] is operative. In a theory with a conserved charge (global or local), a superselection rule operates among sectors of distinct charge values because the conservation of charge is associated with the inobservability of rescaling operation $\Psi \to e^{iQ}\Psi$. For the case of the Dirac fermions, the gauge symmetry is broken, however the overall phase corresponding to the fermion number continues to be a symmetry of the effective theory. This permits independent rescaling of the sectors with different values of the fermionic number; thus superselecting bosonic from fermionic sectors. In the case at hand, half-integer values occur, preventing such states from decaying in isolation to the trivial ground state [3] [4].

For the case of majorana fermions where the number operator is not conserved by interactions the validity of such results is far from obvious. However, the occurrence of half-integral values can be shown to be intimately connected to the charge conjugation invariance[11] of the theory. Here we show that in such a case it is possible to assign parities to the lowest energy solitonic sector which are compatible with parity assignment in the vacuum sector. A superselection rule can then be proved for majorana fermions.

In the following we begin with constructing explicit examples of cosmic strings which are metastable and have odd number of zero-modes, for both the cases of Dirac and Majorana masses. The masses in each case are derived from spontaneous symmetry breaking. We then discuss assignment of fermion number to topological objects, including the need for a finite spectrum of negative values for states of trapped majorana fermions. Subsequently we derive the required superselection rule. The paper ends with summary and conclusion.

EXAMPLES

Here we construct two examples in which the topological objects of a low energy theory are metastable due to the embedding of the low energy symmetry group in a larger symmetry group at higher energy. Examples of this kind were considered in [12]. Borrowing the strategies for bosonic sector from there, we include appropriate fermionic content to ensure the zero-modes.

A Dirac fermions

Consider first a model with two stages of symmetry breaking similar to [12], but with local $SU(2)$ gauge invariance. The two scalars $\vec{\Sigma}$ and σ are respectively real triplet and complex doublet. The Lagrangian is taken to be

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4}F^{\mu\nu a}F^a_{\mu\nu} + \frac{1}{2}D_\mu\vec{\Sigma}\cdot D^\mu\vec{\Sigma} + D_\mu\sigma^\dagger D^\mu\sigma \\ &- \lambda_1(\vec{\Sigma}\cdot\vec{\Sigma}-\eta_1^2)^2 - \lambda_2(\sigma^\dagger\sigma-\eta_2^2)^2 \\ &+ \lambda_{12}\eta_1\vec{\Sigma}\cdot\sigma^\dagger\vec{\tau}\sigma \end{aligned} \quad (1)$$

where isovector notation is used and τ are the Pauli matrices. It is assumed that $\eta_1 \gg \eta_2$ and that the coupling $\lambda_{12} > 0$ satisfies $\lambda_{12}\eta_1^2 \ll \lambda_2\eta_2^2$

The vacuum expectation value (VEV) $\vec{\Sigma} = (0\;0\;\eta_1)^T$ breaks the $SU(2)$ to the $U(1)$ generated by $\exp(i\tau^3\alpha/2)$. The effective theory of the σ can be rewritten as the theory of two complex scalar σ_u and σ_d for the up and the down components respectively. The potential of the effective theory favors the minimum

$$\sigma_u = \eta_2, \qquad \sigma_d = 0 \tag{2}$$

In the effective theory σ_u enjoys a $U(1)$ invariance $\sigma_u \to e^{i\alpha}\sigma_u$ which is broken by the above VEV to $\mathbb{Z} \equiv \{e^{2n\pi i}\}$, $n = 1, 2, \ldots$. This makes possible vortex solutions with an ansatz in the lowest winding number sector

$$\sigma_u(r, \phi) = \eta_2 f(r) e^{-i\phi} \tag{3}$$

with r, ϕ planar coordinates with vortex aligned along the z axis. The vortex configuration is a local minimum, however it can decay by spontaneous formation of a monopole-antimonopole pair [12]. These monopoles are permitted by the first breaking $SO(3) \to SO(2)$ in the Σ sector. Paraphrasing the discussion of [12], we have $SU(2) \to U(1) \to \mathbb{Z}$. The vortices are stable in the low energy theory because $\pi_1(U(1)/\mathbb{Z})$ is nontrivial. But in the $SU(2)$, the $\mathbb{Z}$ lifts to $\{e^{4n\pi i\tau^3/2}\} = I$ making it possible to unwind the vortex by crossing an energy barrier.

Consider now the introduction of a doublet of fermion species $\psi_L \equiv (N_L, E_L)$ assumed to be left handed and a singlet right handed species N_R. The Yukawa coupling of these to the σ_u is given by $h\overline{N_R}\sigma^\dagger\psi_L$, which in the vortex sector reads

$$\mathcal{L}_{\sigma-\psi} \sim h\eta_2 f(r)(e^{-i\phi}\overline{N_R}N_L + h.c.) \tag{4}$$

The lowest energy bound states resulting from this coupling are characterized by a topological index, [13] $\mathcal{I} \equiv n_L - n_R$ where n_L and n_R are the zero modes of the left handed and the right handed fermions respectively. This index can be computed using the formula [13][14]

$$\mathcal{I} = \frac{1}{2\pi i}(\ln \det M)|_{\phi=0}^{2\pi} \tag{5}$$

where M is the position dependent effective mass matrix for the fermions. In the present case this gives rise to a single zero-energy mode for the fermions of species N. According to well known reasoning [1] to be recapitulated below, this requires the assignment of either of the values $\pm 1/2$ to the fermion number of this configuration.

B Majorana fermions

The example above can be extended to the case where the N is a majorana fermion. Being a singlet N admits a mass term $M_M\overline{N_R^C}N_R$, M_M signifying majorana mass. This could also be a spontaneously generated mass due to the presence of a neutral scalar χ with coupling terms $h_M\overline{N_R^C}N_R\chi + h.c.$. If this χ acquires a VEV at energies higher than the Σ, the N particles possess a majorana mass and fermion number is not a conserved observable.

Finally we present the case where majorana mass is spontaneously generated at the same scale at which the vortex forms. Consider a theory with local $SU(3)$ symmetry broken to $U(1)$ by two scalars, Φ an octet acquiring a VEV $\eta_1\lambda_3$ (λ_3 here being the third Gell-Mann matrix) and ϕ, a $\overline{3}$, acquiring the VEV $\langle\phi^k\rangle = \eta_2\delta^{k2}$, with $\eta_2 \ll \eta_1$. Thus

$$SU(3) \xrightarrow{8} U(1)_3 \otimes U(1)_8 \xrightarrow{\overline{3}} U(1)_+ \tag{6}$$

Here $U(1)_3$ and $U(1)_8$ are generated by λ_3 and λ_8 respectively, and $U(1)_+$ is generated by $(\sqrt{3}\lambda_8 + \lambda_3)/2$ and likewise $U(1)_-$ to be used below. It can be checked that this pattern of VEVs can be generically obtained from the quartic scalar potential of the above Higgses. The effective theory at the second breaking $U(1)_- \to \mathbb{Z}$ gives rise to cosmic strings. However the $\mathbb{Z}$ lifts to identity in the $SU(3)$ so that the string can break with the formation of monopole-antimonopole pair.

Now add a multiplet of left-handed fermions belonging to $\overline{15}$. Its mass terms arise from the following coupling to the $\overline{3}$

$$\mathcal{L}_{\text{Majorana}} = h_M\overline{\psi^C}{}_k^{\{ij\}}\psi_n^{\{lm\}}\phi^r(\varepsilon_{ilr}\delta_j^n\delta_m^k) \tag{7}$$

The indices symmetric under exchange have been indicated by curly brackets. No mass terms result from the 8 because it cannot provide a singlet from tensor product with $\overline{15} \otimes \overline{15}$ [15]. After substituting the ϕ VEV a systematic enumeration shows that all but the two components $\psi_1^{\{22\}}$ and $\psi_3^{\{22\}}$ acquire majorana masses at the second stage of the breaking. Specifically we find the majorana mass matrix to be indeed rank 13. Thus, using either of the results [16] or [14] i.e., eq. (5) we can see that there will be 13 zero modes present in the lowest winding sector of the cosmic string. Thus the induced fermion number differs from that of the vacuum by half-integer as required.

C Final state zero-modes

The stability argument being advanced is in jeopardy if the final state after rupture of the topological object also possesses half-integral fermionic charge. To see that this is not the case it is necessary to study the zero-modes on the two semi-infinite strings shown in fig. 1. Generically we expect each of the halves to support the same number of zero-modes, making the total fermion number of the putative final state integer valued, as required for the validity of our argument.

Consider the ansatz for the lower piece (l) with origin at the corresponding monopole and coordinates (r_l, θ_l, ϕ). For the domain $z < 0$ let the ansatz for the field σ be

$$U_l^\infty(\theta_l, \phi)\begin{pmatrix}0\\ \eta_2\end{pmatrix} f_l(r_l) \equiv \exp\{\frac{i}{2}\theta_l\vec{\tau}\cdot\hat{\phi}\}\begin{pmatrix}0\\ \eta_2\end{pmatrix} f_l(r_l) \tag{8}$$

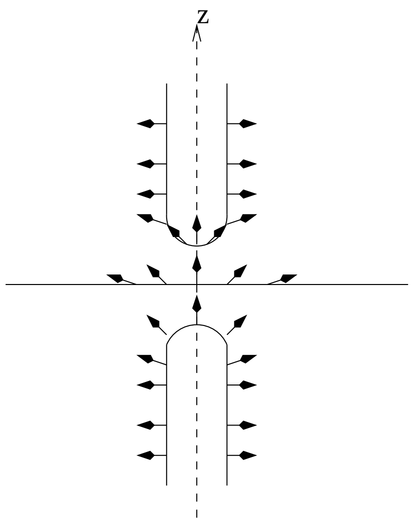

Figure 1: Schematic configuration of isospin vectors after the rupture of a string. Internal orientations are mapped to external space. They are shown just outside the core of the two resulting pieces and on the mid-plane symmetrically separating the two.

so that $\langle\sigma\rangle$ has the behaviour

$$\langle\sigma\rangle = \begin{cases} \begin{pmatrix} 0 \\ \eta_2 \end{pmatrix} f_l(r_l) & \text{for } \theta_l \approx 0 \\ \\ \begin{pmatrix} \eta_2 e^{-i\phi} \\ 0 \end{pmatrix} f_l(r_l) & \text{for } \theta_l \approx \pi \end{cases}$$

which agrees with the ansatz (3) for the cosmic string at the South pole. Likewise for the domain $z > 0$, ie the upper piece (u), we choose

$$U_u^\infty(\theta_u, \phi) = \exp\{\frac{i}{2}(\pi - \theta_u)\vec{\tau}\cdot\hat{\phi}\} \tag{9}$$

resulting in the behaviour

$$\langle\sigma\rangle = \begin{cases} \begin{pmatrix} \eta_2 e^{-i\phi} \\ 0 \end{pmatrix} f_u(r_u) & \text{for } \theta_u \approx 0 \\ \\ \begin{pmatrix} 0 \\ \eta_2 \end{pmatrix} f_u(r_u) & \text{for } \theta_u \approx \pi \end{cases}$$

thus matching correctly with the cosmic string at the North pole. The ansatz for the heavier scalar Σ needs to be appropriately set up, $U^\infty\vec{\Sigma}\cdot\vec{\tau}U^{\infty\dagger}$ in both l and u domains. This scalar however does not contribute to fermion mass matrix.

The two maps match at the mid-plane where $\theta_l = \pi - \theta_u$ and $\sigma_u \sim e^{-i\phi}$ at $\theta_l = \theta_u = \pi/2$ so that we have ensured that the combined map is within the same homotopy class as the string we began with. Finally, as the two pieces move far away, each can be seen to have the same number of zero-modes. To see this we can choose [17][18] fermion ansatz for the zero-modes compatible with the scalar field ansatz, in each of the patches l and u. In (isospin)$\otimes$(two component spinor) notation for ψ_L and for the two component fermion N_R,

$$\psi_L = U^\infty(\theta,\phi)\begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} \varphi_1(r) \\ \chi_1(r) \end{pmatrix} \qquad N_R = \begin{pmatrix} \varphi_2(r) \\ \chi_2(r) \end{pmatrix} \tag{10}$$

where the labels l, u have been dropped. To analyse the asymptotic radial dependence choose $\gamma^r = \sigma^2$ the Pauli matrix. In each patch one finds the pair $\varphi_1(r), \chi_2(r) \sim e^{-h\eta_2 r}$ to constitute the zero-mode while for the other pair, $\varphi_2(r), \chi_1(r) \sim e^{+h\eta_2 r}$ which are therefore not normalizable. In any case, since each of the pieces acquires the same number of zero-modes, the total fermion number of the putative final state has been proved to be integer as required.

ASSIGNMENT OF FERMION NUMBER

We now recapitulate the reasoning behind the assignment of fractional fermion number. We focus on the Majorana fermion case, which is more nettlesome, while the treatment of the Dirac case is standard [1][2]. In the prime example in $3+1$ dimensions of a single left-handed fermion species Ψ_L coupled to an abelian Higgs model according to

$$\mathcal{L}_\psi = i\overline{\Psi_L}\gamma^\mu D_\mu \Psi_L - \frac{1}{2}(h\phi\overline{\Psi_L^C}\Psi_L + h.c.) \tag{11}$$

the following result has been obtained[16]. For a vortex oriented along the z-axis, and in the winding number sector n, the fermion zero-modes are of the form

$$\psi_0(\mathbf{x}) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\left[U(r)e^{il\phi} + V^*(r)e^{i(n-1-l)\phi}\right] g_l(z+t) \tag{12}$$

In the presence of the vortex, τ^3 (here representing Lorentz transformations on spinors) acts as the matrix which exchanges solutions of positive frequency with those of negative frequency. It is therefore identified as the "particle conjugation" operator. In the above ansatz, the ψ in the zero-frequency sector are charge self-conjugates, $\tau^3\psi = \psi$, and have an associated left moving zero mode along the vortex. The functions satisfying $\tau^3\psi = -\psi$ are not normalizable. The situation is reversed when the winding sense of the scalar field is reversed, ie, for $\sigma_u \sim e^{-in\phi}$. In the winding number sector n, regular normalizable solutions[16] exist for for $0 \le l \le n-1$. The lowest energy sector of the vortex is now 2^n-fold degenerate, and each zero-energy mode needs to be interpreted as contributing a value $\pm 1/2$ to the total fermion number of the individual states[1]. This conclusion is difficult to circumvent if the particle spectrum is to reflect the charge conjugation symmetry of the theory [11]. The lowest possible value of the induced number in this sector is $-n/2$. Any general state of the system is built from one of these states by additional integer number of fermions. All the states in the system therefore possess half-integral values for the fermion number if n is odd.

One puzzle immediately arises, what is the meaning of negative values for the fermion number operator for *Majorana*

fermions? In the trivial vacuum, we can identify the Majorana basis as

$$\psi \;=\; \frac{1}{2}(\Psi_L + \Psi_L^C) \tag{13}$$

This leads to the Majorana condition which results in identification of particles with anti-particles according to

$$\mathcal{C}\psi\mathcal{C}^\dagger \;=\; \psi \tag{14}$$

making negative values for the number meaningless. Here $\mathcal{C}$ is the charge conjugation operator. We shall first verify that in the zero-mode sector we must indeed assign negative values to the number operator. It is sufficient to treat the case of a single zero-mode, which generalizes easily to any larger number of zero-modes. The number operator possesses the properties

$$[N,\psi] \;=\; -\psi \qquad \text{and} \qquad [N,\psi^\dagger] \;=\; \psi^\dagger \tag{15}$$

$$\mathcal{C}N\mathcal{C}^\dagger \;=\; N \tag{16}$$

Had it been the Dirac case, there should be a minus sign on the right hand side of eq. (16). This is absent due to the Majorana condition. The fermion field operator for the lowest winding sector is now expanded as

$$\psi \;=\; c\psi_0 + \left\{ \sum_{\kappa,s} a_{\kappa,s}\chi_{\kappa,s}(x) + \sum_{\mathbf{k},s} b_{\mathbf{k},s}u_{\mathbf{k},s}(x) \;+\; h.c. \right\} \tag{17}$$

where the first summation is over all the possible bound states of non-zero frequency with real space-dependence of the form $\sim e^{-\kappa\cdot\mathbf{x}_\perp}$ in the transverse space directions $\mathbf{x}_\perp$, and the second summation is over all unbound states, which are asymptotically plane waves. These summations are suggestive and their exact connection to the Weyl basis mode functions [19] are not essential for the present purpose. Note however that no "*h.c.*" is needed for the zero energy mode which is self-conjugate. Then the Majorana condition (14) requires that we demand

$$\mathcal{C}\; c\; \mathcal{C}^\dagger \;=\; c \qquad \text{and} \qquad \mathcal{C}\; c^\dagger\; \mathcal{C}^\dagger \;=\; c^\dagger \tag{18}$$

Unlike the Dirac case, the c and $c^\dagger$ are not exchanged under charge conjugation. The only non-trivial irreducible realization of this algebra is to require the existence of a doubly degenerate ground state with states $|-\rangle$ and $|+\rangle$ satisfying

$$c|-\rangle \;=\; |+\rangle \qquad \text{and} \qquad c^\dagger|+\rangle \;=\; |-\rangle \tag{19}$$

with the simplest choice of phases. Now we find

$$\mathcal{C}\; c\; \mathcal{C}^\dagger\mathcal{C}|-\rangle \quad = \mathcal{C}|+\rangle \tag{20}$$

$$\Rightarrow\;\; c(\mathcal{C}|-\rangle) \;=\; (\mathcal{C}|+\rangle) \tag{21}$$

This relation has the simplest non-trivial solution

$$\mathcal{C}|-\rangle \;=\; \eta_C^-|-\rangle \qquad \text{and} \qquad \mathcal{C}|+\rangle \;=\; \eta_C^+|+\rangle \tag{22}$$

where, for the consistency of (19) and (21) η_C^- and η_C^+ must satisfy

$$(\eta_C^-)^{-1}\eta_C^+ \;=\; 1 \tag{23}$$

Finally we verify that we indeed get values $\pm 1/2$ for N. The standard fermion number operator which in the Weyl basis is

$$N_F = \frac{1}{2}[\Psi_L^\dagger\Psi_L - \Psi_L\Psi_L^\dagger] \tag{24}$$

acting on these two states gives,

$$\frac{1}{2}(c\; c^\dagger \;-\; c^\dagger\; c)\;|\pm\rangle \;=\; \pm\frac{1}{2}|\pm\rangle \tag{25}$$

The number operator indeed lifts the degeneracy of the two states. For s number of zero modes, the ground state becomes 2^s-fold degenerate, and the fermion number takes values in integer steps ranging from $-s/2$ to $+s/2$. For s odd the values are therefore half-integral. Although uncanny, these conclusions accord with some known facts. They can be understood as spontaneous symmetry breaking for fermions[20]. The negative values of the number thus implied occur only in the zero-energy sector and do not continue indefinitely to $-\infty$. Instead of an unfathomable *Dirac sea* we have a small *Majorana pond* at the threshold.

QUANTUM MECHANCIAL STABILITY

The theory of eq. (1) possesses a gauge symmetry which is reflected in the effective theory (4) as $N_L \to e^{i\alpha}N_L$, $N_R \to e^{i\alpha}N_R$ giving rise to the usual conserved number for Dirac fermions. The lowest winding vortex sector results in half-integer values for this number. Quantum Mechanical stability of this sector follows from well known arguments [9][10] which can now be understood as either following from distinctness of sectors of different values of $(-1)^{N_F}$, or as a consequence of a residual subgroup of the gauge symmetry. For the Majorana case we shall now carry out this kind of argument explicitly.

It is known that Majorana fermions can be assigned a unique parity [10], either of the values $\pm i$. Accordingly let us choose i to be the parity of the free single fermion states in the trivial vacuum. As a step towards deriving our superselection rule, we determine the parities of the zero-energy states. The fermion spectrum should look the same as trivial vacuum far away from the vortex [24]. In turn the parities of the latter states should be taken to be the same as those of the trivial ground state. Next, any of these asymptotic free fermions is capable of being absorbed by the vortex (see for instance [21]). In the zero energy sector this absorption would cause a transition from $|-1/2\rangle$ to $|1/2\rangle$ and cause a change in parity by i. Thus the level carrying fermion number $+1/2$ should be assigned a parity $e^{i\pi/2}$ relative to the $-1/2$ state. Symmetry between the two states suggests that we assign parity $e^{i\pi/4}$ to the $N_F = 1/2$ and $e^{-i\pi/4}$ to the $N_F = -1/2$ states.

Similar reasoning applies to a residual discrete symmetry belonging to the original $U(1)$ gauge group of Lagrangian (11). According to eq. (13), under gauge transformation,

$$\psi \to \psi_{[\alpha]} \equiv \frac{1}{2}(e^{i\alpha}\Psi_L + e^{-i\alpha}\Psi_L^C) \tag{26}$$

Thus $\alpha = \pi$ preserves the choice of the Majorana basis upto a sign. After symmetry breakdown and Higgs mechanism, the Yukawa coupling takes the form $\sim (m+\tilde{\phi})\overline{\psi}\psi$, which is invariant under the residual $\mathbb{Z}_2$ symmetry $\psi \to -\psi$. We can use this as a discrete symmetry distinguishing states of even and odd majorana fermions. Since single majorana fermions can be absorbed by the vortex [21], the ground states $|\pm\rangle$ are distinguished from each other by a relative negative sign. To be symmetric we can assign the value $\pm i$ to these states under this discrete symmetry with sign same as in the value of the number operator. It is possible to prove the superselection rule using this conserved quantity. However we also see that this discrete symmetry can be used to change our convention of the parity for free majorana particles from $+i$ to $-i$. Thus the two are intimately related and in what follows we shall use the parity with convention as in the preceding paragraph.

We now show the inappropriateness of superposing states of half-integer valued fermion number and integer valued fermion number [9]. The operation $\mathcal{P}^4$, parity transformation performed four times must return the system to the original state, upto a phase. Consider forming the state $\Psi_S = \frac{1}{\sqrt{2}}(|1/2\rangle + |1\rangle)$ from states of half-integer and integer value for the fermion number. But

$$\mathcal{P}^4\,\Psi_S \;=\; \frac{1}{\sqrt{2}}(-|1/2\rangle + |1\rangle) \tag{27}$$

Thus this operation identifies a state with another orthogonal to it. Similarly, application of $\mathcal{P}^2$ which should also leave the physical content of a state unchanged results in yet another linearly independent state, $\frac{1}{\sqrt{2}}(i|1/2\rangle - |1\rangle)$. Thus the space of superposed states collapses to a trivial vector space. The conclusion therefore is that it is not possible to superpose such sectors. In turn there can be no meaningful operator possessing non-trivial matrix elements between the two spaces. This completes our proof of the theorem.

CONCLUSION

Metastable classical lumps, also referred to as embedded defects can be found in several theories. The conditions on the geometry of the vacuum manifold that give rise to such defects were spelt out in [12]. We have studied the related question of fermion zero-energy modes on such objects. It is possible to construct examples of cosmic strings in which the presence of zero-modes signals a fractional fermion number both for Dirac and Majorana masses. It then follows that such a cosmic string cannot decay in isolation because it belongs to a distinct superselected Quantum Mechanical sector. Thus a potentially metastable object can enjoy induced stability due to its bound state with fermions.

Although decay is not permitted in isolation, it certainly becomes possible when more than one such objects come together in appropriate numbers. In the early Universe such objects could have formed depending on the unifying group and its breaking pattern. Their disappearance would be slow because it can only proceed through encounters between objects with complementary fermion numbers adding up to an integer. Another mode of decay is permitted by change in the ground state in the course of a phase transition. When additional Higgs fields acquire a vacuum expectation value, in turn altering the boundary conditions for the Dirac or Majorana equation, the number of induced zero modes may change from being odd to even[25], thus imparting the strings an integer fermion number. The decay can then proceed at the rates calculated in [12].

In condensed matter systems the best known example is of charge carriers with half the electronic charge in polyacetylene [23] where the underlying solitons have been identified as stable kinks. It would be interesting to find intrinsically metastable objects stabilized by this mechanism.

Finally it is interesting to conjecture about the fate of loops of cosmic strings. The zero modes on closed string loops have not been studied in detail, though dynamical stability due to superconductivity has been identified. If loops carry fractional fermion number and get stabilized by this mechanism that also would present very interesting possibilities for Cosmology.

ACKNOWLEDGEMENT

We thank Sumit Das, P. Ramadevi and S. Uma Sankar for their for critical comments on the manuscript. We are also indebted to the referee for critical comments.

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